

Angle Relationships and Triangles

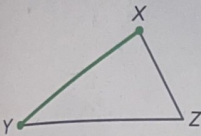
I Can... find the measures of interior and exterior angles in a triangle by using relationships between these angles.

Learn Triangles

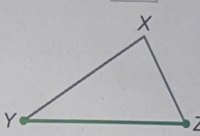
A **line segment** is part of a line containing two endpoints and all of the points between them. A triangle is formed by three line segments that intersect at their endpoints. A point where the segments intersect is a **vertex**. The three angles that lie inside a triangle, formed by the segments and the vertices, are called **interior angles**.

Triangle XYZ, written $\triangle XYZ$, has sides and angles that can be named using its vertices X, Y, and Z. The angle located at vertex Y can be named with symbols as $\angle Y$, $\angle XYZ$, or $\angle ZYX$. The sides of a triangle can be named using segment notation. For example, XY is read as *segment XY*. Name the missing sides, vertices, and angles.

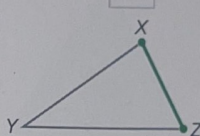
Side: \overline{XY}



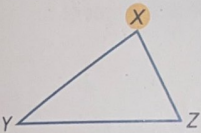
Side: \overline{YZ}



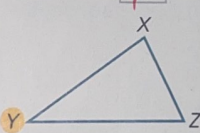
Side: \overline{XZ}



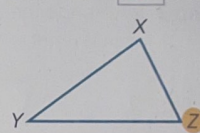
Vertex: X



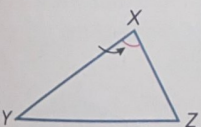
Vertex: Y



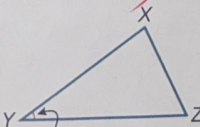
Vertex: Z



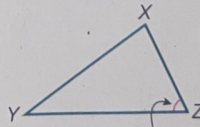
Angle: $\angle X$



Angle: $\angle Y$



Angle: $\angle Z$



What Vocabulary

Will You Learn?

exterior angle

interior angles

line segment

remote interior angles

vertex

أجزاء المثلث
التكليم

Mrs / Aya

05017175602

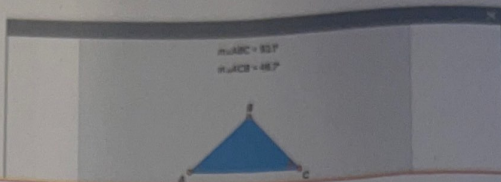
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Mrs / Aya

050/7175602

Explore Angles of Triangles

Online Activity You will use Web Sketchpad to explore the relationship among the angle measures in triangles.



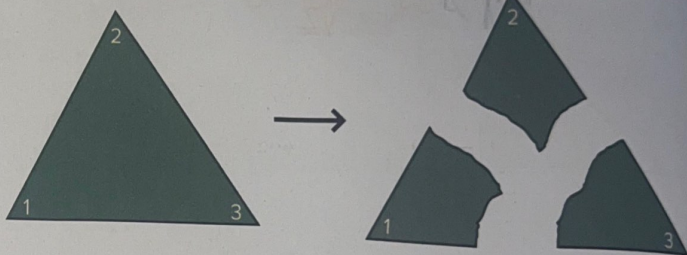
Learn Angle Sum of Triangles

The measures of the angles in a triangle have a special relationship.

Go Online Watch the video and follow these steps to learn about the relationship among the angles in a triangle.

Step 1 Draw a triangle like the one shown below.

Step 2 Tear off each corner.



Step 3 Rearrange the torn pieces so that the corners all meet at one point.



The angles form a straight line. This means the sum of their measures is 180° . Consider another triangle, shown below. Complete the equation for the sum of the measures of the angles.

$$110^\circ + \boxed{45} + \boxed{25} = \boxed{180}$$



(continued on next page)

The activity on the previous page illustrates the relationship among the angle measures of a triangle.

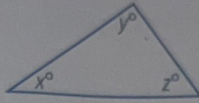
Words

The sum of the measures of the interior angles of a triangle is 180° .

Variables

$$x + y + z = 180$$

Model



ایصال المنطق

Mrs/ Aya

05017175602

Example 1 Find Missing Angle Measures

Find the value of x in the flag of Saint Kitts and Nevis.



$$x + 56 + 90 = 180$$

Write the equation.

$$x + 146 = 180$$

Add.

$$-146 = -146$$

Subtraction Property of Equality

$$x = 34$$

Simplify.

So, the value of x in the triangle is 34.

Check

What is the value of x in the doghouse shown?

Show your work here

$$x + 86 + 47 = 180$$

$$x + 133 = 180$$

$$-133 \quad -133$$

$$x = 47$$



Go Online You can complete an Extra Example online.

Think About It!
Can the measure of Angle A be 30° ? Explain.

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Example 2 Use Ratios to Find Angle Measures

In $\triangle ABC$, the measures of the angles A, B, and C, respectively, are in the ratio 1 : 4 : 5.

Find the measure of each angle.

Step 1 Write an equation.

| Words |
|---|
| The sum of the angle measurements in a triangle is 180° . |
| Variable |
| Let x represent the measure of angle A. |
| The measure of angle B is 4 times greater, or $4x$. The measure of angle C is 5 times greater than x , or $5x$. |
| Equation |
| $x + 4x + 5x = 180$ |

Step 2 Solve the equation and evaluate the angle measurements.

| | |
|---------------------|---------------------|
| $x + 4x + 5x = 180$ | Write the equation. |
| $10x = 180$ | Combine like terms. |
| $x = 18$ | Simplify. |

Since $x = 18$, $m\angle A$ is 18° . The measure of $\angle B$ is $4x^\circ$, or $4(18)$, which is 72° . The measure of $\angle C$ is $5x^\circ$, or $5(18)$, which is 90° .

Check

In $\triangle LMN$, the measures of the angles L, M, and N, respectively, are in the ratio 1 : 2 : 5. Find the measure of each angle.

Show your work here
 $1x + 2x + 5x = 180$
 $8x = 180$
 $x = 22.5$

$1 \times 22.5 = 22.5$ $2 \times 22.5 = 45$ $5 \times 22.5 = 112.5$
 $m\angle L = 22.5$ $m\angle M = 45$ $m\angle N = 112.5$

$22.5 + 45 + 112.5 = 180$

Go Online You can complete an Extra Example online.

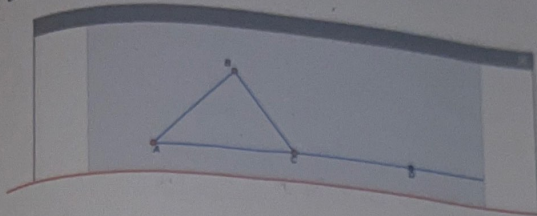
Talk About It!
How can you check your solution?

Add the measures of the angles

$18 + 72 + 90 = 180$

Explore Exterior Angles of Triangles

Online Activity You will use Web Sketchpad to explore the relationship between an exterior angle and two remote interior angles of a triangle.



Learn Exterior Angles of Triangles

In addition to its three interior angles, a triangle can have an **exterior angle** formed by one side of the triangle and the extension of the adjacent side.

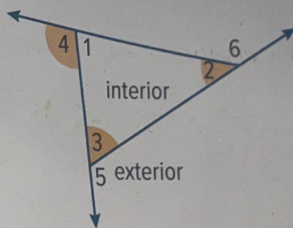
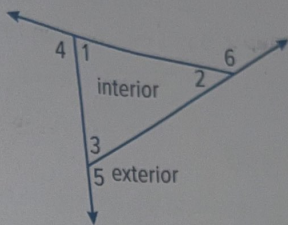
In the diagram shown, angles 4, 5, and 6 are exterior angles. An exterior angle is supplementary to its adjacent interior angle because the two angles form a straight line.

Complete the statements relating each exterior angle and its adjacent interior angle.

$$m\angle 4 + m\angle 1 = 180^\circ$$

$$m\angle 6 + m\angle 2 = 180^\circ \quad m\angle 5 + m\angle 3 = 180^\circ$$

Each exterior angle of the triangle has two **remote interior angles** that are *not* adjacent to the exterior angle. Angle 4 is an exterior angle of the triangle. Its two remote interior angles are $\angle 2$ and $\angle 3$.



Which angles are remote interior angles in relation to $\angle 5$? $\angle 1$

and $\angle 2$

Which angles are remote interior angles in relation to $\angle 6$? $\angle 1$

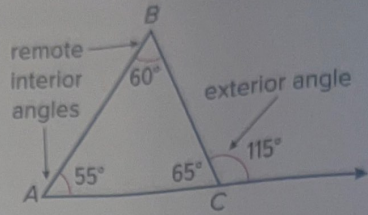
and $\angle 3$

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Go Online Watch the video to learn about the relationship between an exterior angle of a triangle and its two remote interior angles.

The video demonstrates this relationship using triangle ABC.



$$60^\circ + 55^\circ + 65^\circ = 180$$

Angle sum of a triangle

$$65^\circ + 115^\circ = 180$$

Supplementary angles

$$60^\circ + 55^\circ + 65^\circ = 65^\circ + 115^\circ$$

Write the equation.

$$- 65^\circ = - 65^\circ$$

Subtract 65 from each side.

$$60 + 55 = 115$$

Simplify.

Talk About It!

Can the measure of an exterior angle be less than or equal to either of its remote interior angles?

No

Because the measure of an exterior angles the sum of its 2 Remote interior angles.

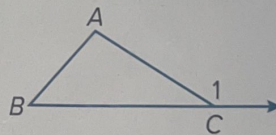
the measure of an exterior angle must be greater than either of its 2 remote interior angles

The equation shows that the sum of the measures of $\angle A$ and $\angle B$ is equal to the measure of the exterior angle.

Words

The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Model



Symbols

$$m\angle A + m\angle B = m\angle 1$$

Example 3 Find Exterior Angle Measures

In the beach chair shown,
 $m\angle 2 = 55^\circ$ and $m\angle 3 = 60^\circ$.
 Find the measure of $\angle 1$.



Angle 1 is an exterior angle. Its two remote interior angles are $\angle 2$ and $\angle 3$.

$$m\angle 2 + m\angle 3 = m\angle 1$$

Write the equation.

$$55 + 60 = m\angle 1$$

$$m\angle 2 = 55^\circ, m\angle 3 = 60^\circ$$

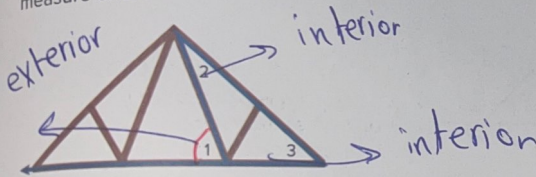
$$115 = m\angle 1$$

Simplify.

So, the measure of $\angle 1$ is 115° .

Check

In the roof frame shown, $m\angle 2 = 25^\circ$ and $m\angle 3 = 45^\circ$. Find the measure of $\angle 1$.



Show your work here

$$m\angle 1 = \angle 2 + \angle 3$$

$$m\angle 1 = 25 + 45$$

$$\angle 1 = 70$$

Go Online You can complete an Extra Example online.

Think About It!

What steps do you need to take to find $m\angle 1$?

Talk About It!

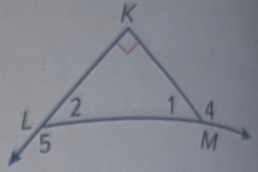
What is another way to find the measure of angle 1?

The sum of the measures of angles 2 and 3 could be subtracted from 180 to find the measure of the third interior angle and angle 1 form a line so, they are supplementary the measure of the third interior angle can be subtracted from 180 to find measure of angle 1

Example 4 Use Exterior Angles to Find Missing Angle Measures

In the figure, $m\angle 4 = 135^\circ$.

Find the measures of $\angle 2$ and $\angle 1$.



Angle 4 is an exterior angle. Its two remote interior angles are $\angle 2$ and $\angle LKM$.

$$m\angle 2 + m\angle LKM = m\angle 4$$

$$m\angle 2 + 90 = 135$$

$$m\angle 2 = 45$$

Write the equation.

$$m\angle LKM = 90^\circ \text{ and } m\angle 4 = 135^\circ$$

Subtraction Property of Equality

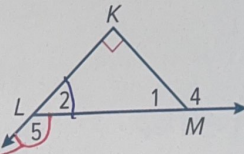
Since $\angle 1$ and $\angle 4$ are supplementary, the sum of their measures is

180.

So, $m\angle 1$ is $180^\circ - 135^\circ$ or 45° .

Check

In the figure, $m\angle 5 = 147^\circ$. Find the measures of $\angle 1$ and $\angle 2$.



147

$$m\angle 5 = \angle K + \angle 1$$

$$m\angle 2 = 180 - 147$$

$$\angle 2 = 33^\circ$$

$$\angle 1 = 180 - (90 + 33)$$

$$180 - 123 = 57$$

Adjacent

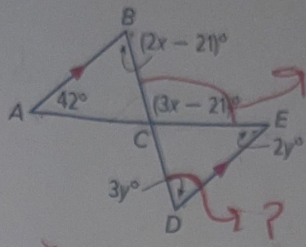
Go Online You can complete an Extra Example online.

Apply Geometry

What are the measures of $\angle CDE$ and $\angle BCE$ in the figure?

exterior = 2 Remote interior

~~$3x - 21 = 2x - 21 + 42$~~
 ~~$3x + 2 = 2x + 21$~~
 ~~$+2$~~
 ~~$+2$~~
 ~~$3x = 2x + 23$~~
 ~~$2x - 2x$~~
 ~~$x =$~~



$\angle BCE = \angle A + \angle B$
 $3x - 21 = 42 + 2x - 21$
 $+21$ $+21$

 $3x = 42 + 2x$
 $-2x$ $-2x$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.

Second Time What mathematics do you see in the problem?

Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

Step 2

$\angle BCE = \angle D + \angle E$
 $105 = 3y + 2y$
 $105 = 5y$

 5

3 What is your solution?

Use your strategy to solve the problem.

$y = 21^\circ$

$\therefore \angle CDE = 21 \times 3 = 63$
 $\angle BCE = 105^\circ, \angle CDE = 63^\circ$

Talk About It!
 Is there more than one way to find the measure of $\angle BCE$? Explain.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Mrs / Aya
 050/7/75602

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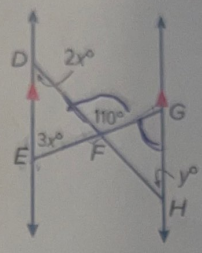
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Check

In the diagram, the two vertical lines are parallel. Find the measures of $\angle FDE$, $\angle DEF$, and $\angle FHG$.



$$\begin{aligned} \angle 110 &= \angle D + \angle E \\ 110 &= 2x + 3x \\ 110 &= 5x \quad x = 22 \\ \frac{110}{5} &= \frac{5x}{5} \end{aligned}$$

Show your work here

$$\angle FDE = 2 \times 22 = 44$$

$$\angle DEF = 2 \times 22 = 66$$

$$\angle FHG = 44 \rightarrow \text{Alternat. interior with } \angle FDE$$

Go Online You can complete an Extra Example online.

Pause and Reflect

Explain what you have learned about the interior angles, exterior angles, and remote interior angles of a triangle. Given a drawing, can you identify these angles?

Record your observations here

Name: إيمان العتيق Period: Mrs/AYG Date: 05/07/2020

Practice

Go Online You can complete your homework online.

Find the value of x in each object. (Example 1)

1.



$$30 + 30 + x = 180$$

$$60 + x = 180$$

$$x = 180 - 60 = 120$$

3. In $\triangle FGH$, the measures of angles F , G , and H , respectively, are in the ratio $4 : 4 : 10$. Find the measure of each angle. (Example 2)

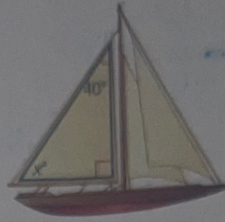
$$4x + 4x + 10x = 180$$

$$\frac{18x}{18} = \frac{180}{18}$$

$$x = 10$$

$\angle F = 4(10) = 40^\circ$
 $\angle G = 4(10) = 40^\circ$
 $\angle H = 10(10) = 100^\circ$

2.

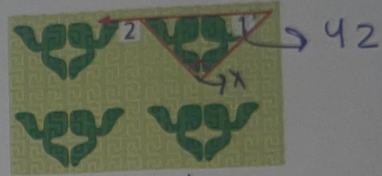


$$40 + 90 + x = 180$$

$$130 + x = 180$$

$$x = 180 - 130 = 50$$

4. In the knitting pattern, $m\angle 1 = 42^\circ$. Find the measure of $\angle 2$. (Example 3)



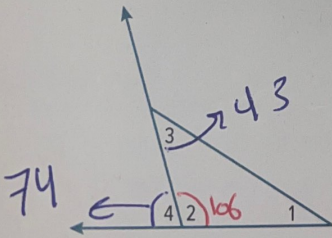
$$\angle 2 = \angle 1 + \angle x$$

$$\angle 2 = 40 + 90 = 132$$

$$\angle 2 = 132$$

Test Practice

5. In the figure, $m\angle 4 = 74^\circ$ and $m\angle 3 = 43^\circ$. Find the measures of $\angle 1$ and $\angle 2$. (Example 4)



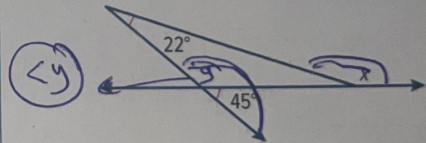
$$\angle 4 = \angle 3 + \angle 1$$

$$\angle 2 = 180 - 74 = 106$$

$$\angle 1 = 180 - (43 + 106)$$

$$\angle 1 = 180 - 149 = 31$$

6. Open Response What is the measure of $\angle x$, in degrees, in the figure shown?



$$\angle x = \angle 22 + \angle y$$

$$\angle y = 180 - 45 = 135$$

$$\angle x = \angle 22 + \angle 135$$

$$= 157^\circ$$

Apply

7. What are the measures of $\angle ADC$ and $\angle DCB$ in the figure below?

$3y + 7 + 3y - 13 = 180$
 $6y = 180 - 14 = 166$
 $y = 31$
 $m\angle ADC = 3y + 7 = 3(31) + 7 = 100$
 $\angle DCB = 180 - (80 + 61) = 39$

8. What are the measures of $\angle CAB$ and $\angle ACB$ in the figure below?

$\angle CAB = (3 \times 11) - 10 = 23$
 $103 = (3x - 10) + (7x + 3)$
 $103 = 10x - 7$
 $110 = 10x$
 $x = 11$

9. **MP Find the Error** A student is finding the measures of the angles in a triangle that have the ratio 4 : 4 : 7. Find the mistake and correct it.

$4x + 4x + 7x = 180$
 $15x = 180$
 $x = 12$

So, the angle measures are 12, 12, and 84.

After finding value of x
 So $4(12) = 48$
 $7(12) = 84$
 So, 3 Angles 48, 48, 84

10. **MP Persevere with Problems** The measure of $\angle A$ in $\triangle ABC$ is twice the measure of $\angle B$, and $\angle C$ is 20° less than the measure of $\angle B$. What are the measures of the angles in $\triangle ABC$?

$m\angle A = 100$
 $m\angle B = 50$
 $m\angle C = 30$

11. Determine if the statement is true or false. Construct an argument that can be used to defend your solution.

An exterior angle of a triangle will always be obtuse.

If angles of a triangle is obtuse, then the exterior angle that is supplementary to it will be acute

12. **MP Find the Error** A student states that the exterior angle of a triangle can never be a right angle. Find the mistake and correct it.

An exterior angle of a triangle can be supplementary to the right triangle so an exterior can be right angle